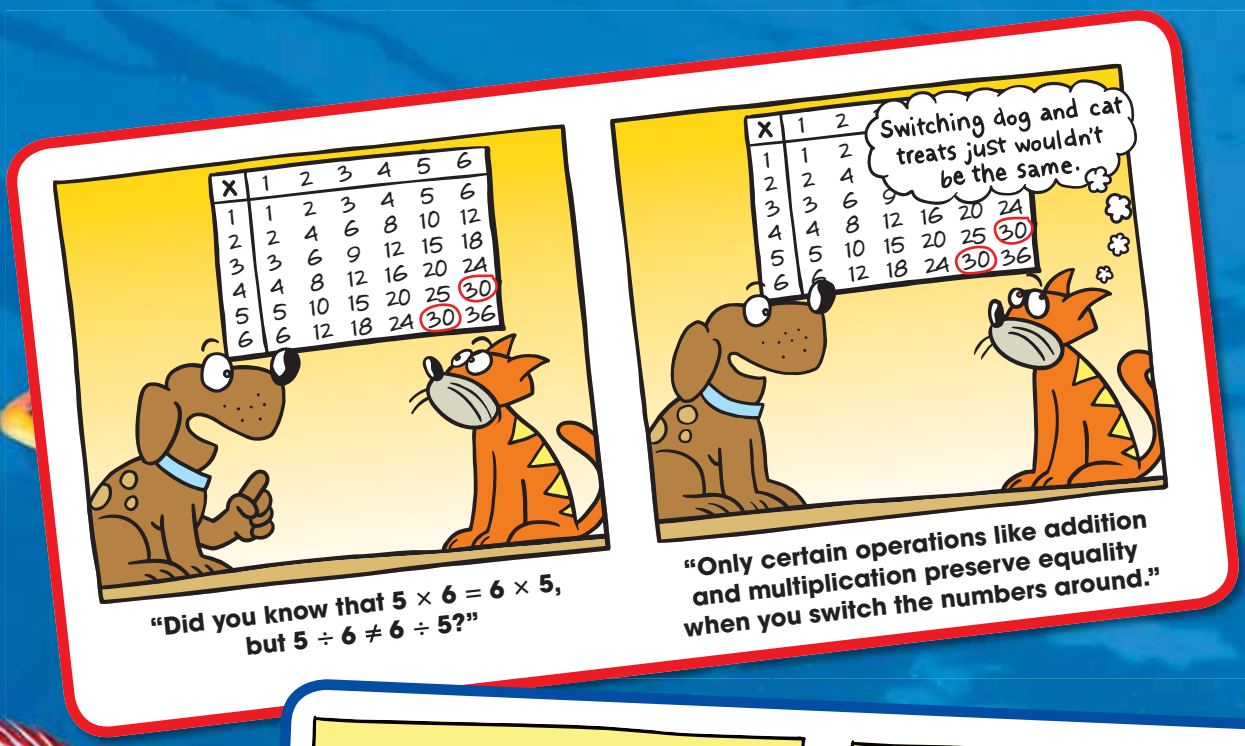
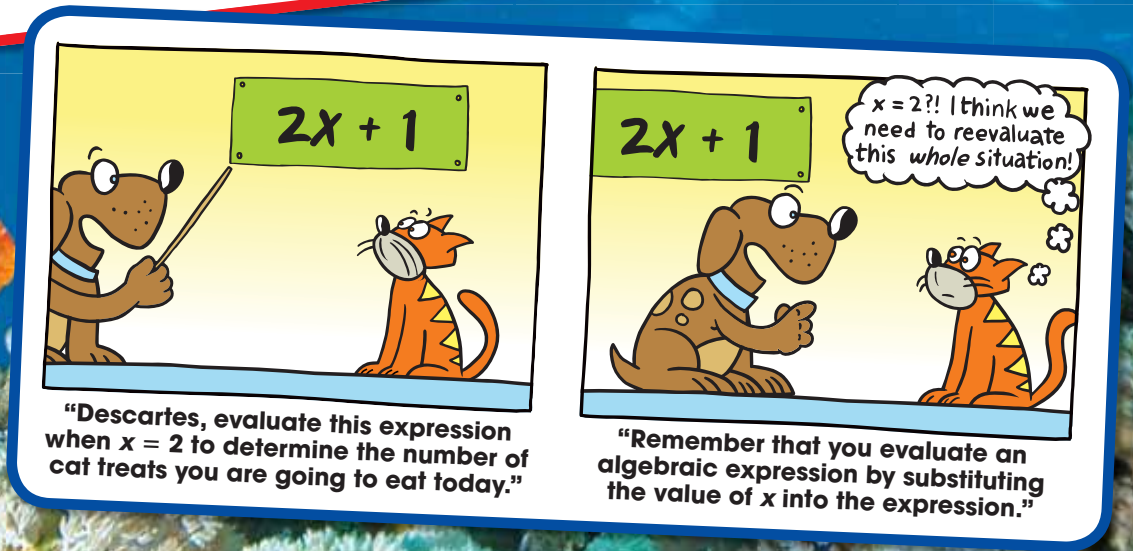
3 Algebraic Expressions and Properties





What You Learned Before

Interpreting Numerical Expressions (5.OA.2)

Example 1 **Write a sentence interpreting the expression $3 \times (19{,}762 + 418)$.**

$3 \times (19{,}762 + 418)$ is 3 times as large as $19{,}762 + 418$.

Example 2 **Write a sentence interpreting the expression $(316 + 43{,}449) + 5$.**

$(316 + 43{,}449) + 5$ is 5 more than $316 + 43{,}449$.

Example 3 **Write a sentence interpreting the expression $(20{,}008 - 752) \div 2$.**

$(20{,}008 - 752) \div 2$ is half as large as $20{,}008 - 752$.

Try It Yourself

Write a sentence interpreting the expression.

1. $3 \times (372 + 20{,}967)$
2. $2 \times (432 + 346{,}322)$
3. $4 \times (6722 + 4086)$
4. $(115 + 36{,}372) + 6$
5. $(392 + 75{,}325) + 78$
6. $(352 + 46{,}795) + 100$
7. $(30{,}929 + 425) \div 2$
8. $(58{,}742 - 721) \div 2$
9. $(96{,}792 + 564) \div 3$

Using Order of Operations (5.OA.1, 6.EE.1)

Example 4 **Simplify $4^2 \div 2 + 3(9 - 5)$.**

First: **P**arentheses
Second: **E**xponents
Third: **M**ultiplication and **D**ivision (from left to right)
Fourth: **A**ddition and **S**ubtraction (from left to right)

$$4^2 \div 2 + 3(9 - 5) = 4^2 \div 2 + 3 \cdot 4$$
$$= 16 \div 2 + 3 \cdot 4$$
$$= 8 + 12$$
$$= 20$$

Try It Yourself

Simplify the expression.

10. $3^2 + 5(4 - 2)$
11. $3 + 4 \div 2$
12. $10 \div 5 \cdot 3$
13. $4(3^3 - 8) \div 2$
14. $3 \cdot 6 - 4 \div 2$
15. $12 + 7 \cdot 3 - 24$

3.1 Algebraic Expressions

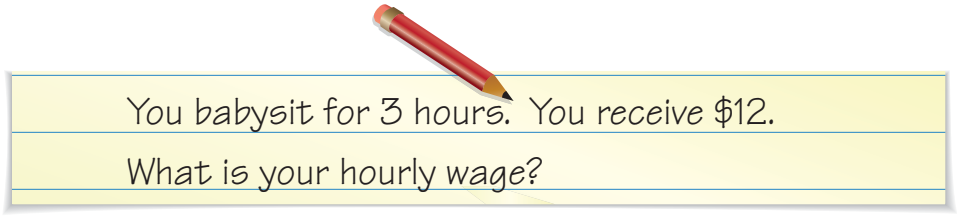
Essential Question How can you write and evaluate an expression that represents a real-life problem?

1 ACTIVITY: Reading and Re-Reading

Work with a partner.

a. You babysit for 3 hours. You receive $12. What is your hourly wage?

- Write the problem. Underline the important numbers and units you need to solve the problem.
- Read the problem carefully a second time. Circle the key word for the question.



- Write each important number or word, with its units, on a piece of paper. Write +, −, ×, ÷, and = on five other pieces of paper.

- Arrange the pieces of paper to answer the key word question, "What is your hourly wage?"
- Evaluate the expression that represents the hourly wage.

hourly wage = ☐ ÷ ☐ Write.

= ☐ Evaluate.

So, your hourly wage is $☐ per hour.

b. How can you use your hourly wage to find how much you will receive for any number of hours worked?

Common Core

Algebraic Expressions

In this lesson, you will

- use order of operations to evaluate algebraic expressions.
- solve real-life problems.

Learning Standard 6.EE.2c

2 ACTIVITY: Reading and Re-Reading

Math Practice 2

Make Sense of Quantities

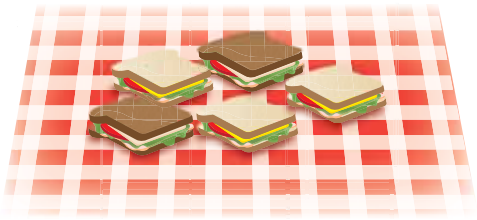
What are the units in the problem? How does this help you write an expression?

Work with a partner. Use the strategy shown in Activity 1 to write an expression for each problem. After you have written the expression, evaluate it using mental math or some other method.

a. You wash cars for 2 hours. You receive $6. How much do you earn per hour?

b. You have $60. You buy a pair of jeans and a shirt. The pair of jeans costs $27. You come home with $15. How much did you spend on the shirt?

c. For lunch, you buy 5 sandwiches that cost $3 each. How much do you spend?



d. You are running a 4500-foot race. How much farther do you have to go after running 2000 feet?

e. A young rattlesnake grows at a rate of about 20 centimeters per year. How much does a young rattlesnake grow in 2 years?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you write and evaluate an expression that represents a real-life problem? Give one example with addition, one with subtraction, one with multiplication, and one with division.

Practice Use what you learned about evaluating expressions to complete Exercises 4–7 on page 115.

3.1 Lesson

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Key Vocabulary
algebraic expression, *p. 112*
terms, *p. 112*
variable, *p. 112*
coefficient, *p. 112*
constant, *p. 112*

An **algebraic expression** is an expression that may contain numbers, operations, and one or more symbols. Parts of an algebraic expression are called **terms**.

A symbol that represents one or more numbers is called a **variable**.

$5p + 4$

The numerical factor of a term that contains a variable is a **coefficient**.

A term without a variable is called a **constant**.

EXAMPLE 1 Identifying Parts of an Algebraic Expression

Identify the terms, coefficients, and constants in each expression.

a. $5x + 13$

$\underbrace{5x}_{} + \underbrace{13}_{}$

Terms: $5x$, 13

Coefficient: 5

Constant: 13

b. $2z^2 + y + 3$

$\underbrace{2z^2}_{} + \underbrace{y}_{} + \underbrace{3}_{}$

Terms: $2z^2$, $1y$, 3

Coefficients: 2, 1

Constant: 3

Study Tip
A variable by itself has a coefficient of 1. So, the term y in Example 1(b) has a coefficient of 1.

On Your Own

Now You're Ready
Exercises 8–13

Identify the terms, coefficients, and constants in the expression.

1. $12 + 10c$ **2.** $15 + 3w + \frac{1}{2}$ **3.** $z^2 + 9z$

EXAMPLE 2 Writing Algebraic Expressions Using Exponents

Write each expression using exponents.

a. $d \cdot d \cdot d \cdot d$

Because d is used as a factor 4 times, its exponent is 4.

∴ So, $d \cdot d \cdot d \cdot d = d^4$.

b. $1.5 \cdot h \cdot h \cdot h$

Because h is used as a factor 3 times, its exponent is 3.

∴ So, $1.5 \cdot h \cdot h \cdot h = 1.5h^3$.

On Your Own

Now You're Ready
Exercises 16–21

Write the expression using exponents.

4. $j \cdot j \cdot j \cdot j \cdot j \cdot j$ **5.** $9 \cdot k \cdot k \cdot k \cdot k \cdot k$

To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

EXAMPLE 3 Evaluating Algebraic Expressions

a. Evaluate $k + 10$ when $k = 25$.

$$k + 10 = 25 + 10 \qquad \text{Substitute 25 for } k.$$
$$= 35 \qquad \text{Add 25 and 10.}$$

b. Evaluate $4 \cdot n$ when $n = 12$.

$$4 \cdot n = 4 \cdot 12 \qquad \text{Substitute 12 for } n.$$
$$= 48 \qquad \text{Multiply 4 and 12.}$$

Study Tip

You can write the product of 4 and n in several ways.

$4 \cdot n$

$4n$

$4(n)$

On Your Own

Now You're Ready
Exercises 25–32

6. Evaluate $24 + c$ when $c = 9$.

7. Evaluate $d - 17$ when $d = 30$.

EXAMPLE 4 Evaluating an Expression with Two Variables

Evaluate $a \div b$ when $a = 16$ and $b = \frac{2}{3}$.

$$a \div b = 16 \div \frac{2}{3} \qquad \text{Substitute 16 for } a \text{ and } \frac{2}{3} \text{ for } b.$$
$$= 16 \cdot \frac{3}{2} \qquad \text{Multiply by the reciprocal of } \frac{2}{3}, \text{ which is } \frac{3}{2}.$$
$$= 24 \qquad \text{Multiply.}$$

On Your Own

Now You're Ready
Exercises 33–36

Evaluate the expression when $p = 24$ and $q = 8$.

8. $p \div q$ **9.** $q + p$

10. $p - q$ **11.** pq

EXAMPLE 5 Evaluating Expressions with Two Operations

a. Evaluate $3x - 14$ when $x = 5$.

$3x - 14 = 3(5) - 14$ Substitute 5 for x.

$= 15 - 14$ Using order of operations, multiply 3 and 5.

$= 1$ Subtract 14 from 15.

b. Evaluate $z^2 + 8.5$ when $z = 2$.

$z^2 + 8.5 = 2^2 + 8.5$ Substitute 2 for z.

$= 4 + 8.5$ Using order of operations, evaluate 2^2.

$= 12.5$ Add 4 and 8.5.

On Your Own

Now You're Ready
Exercises 43–51

Evaluate the expression when $y = 6$.

12. $5y + 1$ **13.** $30 - 24 \div y$ **14.** $y^2 - 7$ **15.** $1.5 + y^2$

EXAMPLE 6 Real-Life Application

You are saving money to buy a skateboard. You begin with \$45 and you save \$3 each week. The expression $45 + 3w$ gives the amount of money you save after w weeks.

a. How much will you have after 4 weeks, 10 weeks, and 20 weeks?

b. After 20 weeks, can you buy the skateboard? Explain.

a. Substitute the given number of weeks for w.

Number of Weeks, w	$45 + 3w$	Amount Saved
4	$45 + 3(4)$	$45 + 12 = \$57$
10	$45 + 3(10)$	$45 + 30 = \$75$
20	$45 + 3(20)$	$45 + 60 = \$105$

b. After 20 weeks, you have \$105. So, you cannot buy the \$125 skateboard.

On Your Own

16. WHAT IF? In Example 6, the expression for how much money you have after w weeks is $45 + 4w$. Can you buy the skateboard after 20 weeks? Explain.

3.1 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

   $2x + 1$ | $5w \cdot c$ | $3(4) + 5$ | $y \div z$

2. **NUMBER SENSE** Which step in the order of operations is first? second? third? fourth?

   Add or subtract from left to right. | Multiply or divide from left to right.

   Evaluate terms with exponents. | Perform operations in parentheses.

3. **NUMBER SENSE** Will the value of the expression $20 - x$ *increase*, *decrease*, or *stay the same* as x increases? Explain.

Practice and Problem Solving

Write and evaluate an expression for the problem.

4. You receive \$8 for raking leaves for 2 hours. What is your hourly wage?
5. Music lessons cost \$20 per week. How much do 6 weeks of lessons cost?
6. The scores on your first two history tests were 82 and 95. By how many points did you improve on your second test?
7. You buy a hat for \$12 and give the cashier a \$20 bill. How much change do you receive?

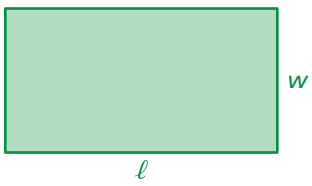
Identify the terms, coefficients, and constants in the expression.

8. $7h + 3$
9. $g + 12 + 9g$
10. $5c^2 + 7d$
11. $2m^2 + 15 + 2p^2$
12. $6 + n^2 + \frac{1}{2}d$
13. $8x + \frac{x^2}{3}$

Terms: 2, x^2, y
Coefficient: 2
Constant: none

14. **ERROR ANALYSIS** Describe and correct the error in identifying the terms, coefficients, and constants in the algebraic expression $2x^2y$.

15. **PERIMETER** You can use the expression $2\ell + 2w$ to find the perimeter of a rectangle where ℓ is the length and w is the width.



   a. Identify the terms, coefficients, and constants in the expression.

   b. Interpret the coefficients of the terms.

Write each expression using exponents.

16. $b \cdot b \cdot b$ **17.** $g \cdot g \cdot g \cdot g \cdot g$ **18.** $8 \cdot w \cdot w \cdot w \cdot w$

19. $5.2 \cdot y \cdot y \cdot y$ **20.** $a \cdot a \cdot c \cdot c$ **21.** $2.1 \cdot x \cdot z \cdot z \cdot z \cdot z$

✗ $3 \cdot n \cdot n \cdot n \cdot n = 4n^3$

22. ERROR ANALYSIS Describe and correct the error in writing the product using exponents.

23. AREA Write an expression using exponents that represents the area of the square.

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

24. ST. IVES Suppose the man in the St. Ives poem has x wives, each wife has x sacks, each sack has x cats, and each cat has x kits. Write an expression using exponents that represents the total number of kits, cats, sacks, and wives going to St. Ives.

ALGEBRA Evaluate the expression when $a = 3$, $b = 2$, and $c = 12$.

25. $6 + a$ **26.** $b \cdot 5$ **27.** $c - 1$ **28.** $27 \div a$

29. $12 - b$ **30.** $c + 5$ **31.** $2a$ **32.** $c \div 6$

33. $a + b$ **34.** $c - a$ **35.** $\frac{c}{a}$ **36.** $b \cdot c$

37. ERROR ANALYSIS Describe and correct the error in evaluating the expression when $m = 8$.

✗
$$\begin{aligned} 5m + 3 &= 5 \cdot 8 + 3 \\ &= 5 \cdot 11 \\ &= 55 \end{aligned}$$

38. LAWNS You earn $15n$ dollars for mowing n lawns. How much do you earn for mowing one lawn? seven lawns?

39. PLANT After m months, the height of a plant is $10 + 3m$ millimeters. How tall is the plant after eight months? three years?

Copy and complete the table.

40.

x	3	6	9
$x \cdot 8$			

41.

x	2	4	8
$64 \div x$			

42. FALLING OBJECT An object falls $16t^2$ feet in t seconds. You drop a rock from a bridge that is 75 feet above the water. Will the rock hit the water in 2 seconds? Explain.

ALGEBRA **Evaluate the expression when $a = 10$, $b = 9$, and $c = 4$.**

5 **43.** $2a + 3$ **44.** $4c - 7.8$ **45.** $\frac{a}{4} + \frac{1}{3}$

46. $\frac{24}{b} + 8$ **47.** $c^2 + 6$ **48.** $a^2 - 18$

49. $a + 9c$ **50.** $bc + 12.3$ **51.** $3a + 2b - 6c$

52. **MOVIES** You rent x new releases and y standard rentals. Which expression tells you how much money you will need?

$3x + 4y$ $4x + 3y$ $7(x + y)$

53. **WATER PARK** You float 2000 feet along a "Lazy River" water ride. The ride takes less than 10 minutes. Give two examples of possible times and speeds. Illustrate the water ride with a drawing.

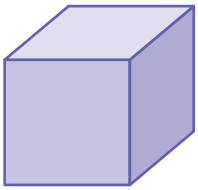
54. **SCIENCE CENTER** The expression $20a + 13c$ is the cost (in dollars) for a adults and c students to enter a science center.

a. How much does it cost for an adult? a student? Explain your reasoning.

b. Find the total cost for 4 adults and 24 students.

c. You find the cost for a group. Then the numbers of adults and students in the group both double. Does the cost double? Explain your answer using an example.

d. In part (b), the number of adults is cut in half, but the number of students doubles. Is the cost the same? Explain your answer.



55. Reasoning The volume of the cube is equal to four times the area of one of its faces. What is the volume of the cube?

Fair Game Review What you learned in previous grades & lessons

Find the value of the power. *(Section 1.2)*

56. 3^5 **57.** 8^3 **58.** 7^4 **59.** 2^8

60. **MULTIPLE CHOICE** Which numbers have a least common multiple of 24? *(Section 1.6)*

(A) 4, 6 (B) 2, 22 (C) 3, 8 (D) 6, 12

3.2 Writing Expressions

Essential Question How can you write an expression that represents an unknown quantity?

1 ACTIVITY: Ordering Lunch

Work with a partner. You use a $20 bill to buy lunch at a café. You order a sandwich from the menu board shown.

a. Complete the table. In the last column, write a numerical expression for the amount of change received.

Sandwich	Price (dollars)	Change Received (dollars)
Reuben		
BLT		
Egg salad		
Roast beef		

b. **REPEATED REASONING** Write an expression for the amount of change you receive when you order any sandwich from the menu board.

c. Compare the expression you wrote in part (b) with the expressions in the last column of the table in part (a).

d. The café offers several side dishes, each at the same price. You order a chicken salad sandwich and two side dishes. Write an expression for the total amount of money you spend. Explain how you wrote your expression.

e. The expression $20 - 4.65s$ represents the amount of change one customer receives after ordering from the menu board. Explain what each part of the expression represents. Do you know what the customer ordered? Explain your reasoning.

Common Core

Algebraic Expressions

In this lesson, you will
- use variables to represent numbers in algebraic expressions.
- write algebraic expressions.

Learning Standard
6.EE.2a

2 ACTIVITY: Words That Imply Addition or Subtraction

Math Practice 2

Use Expressions

How do the key words in the phrase help you write the given relationship as an expression?

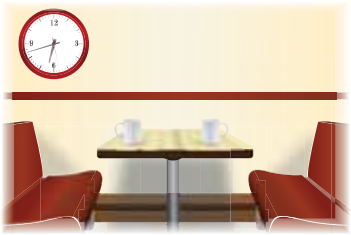
Work with a partner.

a. Complete the table.

Variable	Phrase	Expression
n	4 **more than** a number	
m	the **difference** of a number and 3	
x	the **sum** of a number and 8	
p	10 **less than** a number	
n	7 units **farther** away	
t	8 minutes **sooner**	
w	12 minutes **later**	
y	a number **increased** by 9	

b. Here is a word problem that uses one of the expressions in the table.

You arrive at the café 8 minutes sooner than your friend. Your friend arrives at 6:42 P.M. When did you arrive?



Which expression from the table can you use to solve the problem?

c. Write a problem that uses a different expression from the table.

3 ACTIVITY: Words That Imply Multiplication or Division

Work with a partner. Match each phrase with an expression.

the product of a number and 3	$n \div 3$
the quotient of 3 and a number	$4p$
4 times a number	$n \cdot 3$
a number divided by 3	$2m$
twice a number	$3 \div n$

What Is Your Answer?

4. IN YOUR OWN WORDS How can you write an expression that represents an unknown quantity? Give examples to support your explanation.

Practice Use what you learned about writing expressions to complete Exercises 9–12 on page 122.

3.2 Lesson

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Some words imply math operations.

Operation	Addition	Subtraction	Multiplication	Division
Key Words and Phrases	added to plus sum of more than increased by total of and	subtracted from minus difference of less than decreased by fewer than take away	multiplied by times product of twice of	divided by quotient of

EXAMPLE 1 Writing Numerical Expressions

Write the phrase as an expression.

a. 8 fewer than 21

$21 - 8$ The phrase *fewer than* means *subtraction*.

b. the product of 30 and 9

30×9, or $30 \cdot 9$ The phrase *product of* means *multiplication*.

EXAMPLE 2 Writing Algebraic Expressions

Write the phrase as an expression.

a. 14 more than a number x

$x + 14$ The phrase *more than* means *addition*.

b. a number y minus 75

$y - 75$ The word *minus* means *subtraction*.

c. the quotient of 3 and a number z

$3 \div z$, or $\frac{3}{z}$ The phrase *quotient of* means *division*.

Common Error

When writing expressions involving subtraction or division, order is important. For example, the quotient of a number x and 2 means $x \div 2$, not $2 \div x$.

On Your Own

Now You're Ready
Exercises 3–18

Write the phrase as an expression.

1. the sum of 18 and 35
2. 6 times 50
3. 25 less than a number b
4. a number x divided by 4
5. the total of a number t and 11
6. 100 decreased by a number k

EXAMPLE 3 Writing an Algebraic Expression

The length of Interstate 90 from the West Coast to the East Coast is 153.5 miles more than 2 times the length of Interstate 15 from southern California to northern Montana. Let m be the length of Interstate 15. Which expression can you use to represent the length of Interstate 90?

Ⓐ $2m + 153.5$ Ⓑ $2m - 153.5$ Ⓒ $153.5 - 2m$ Ⓓ $153.5m + 2$

The word *times* means *multiplication*. So, multiply 2 and m.

$2m + 153.5$

The phrase *more than* means *addition*. So, add $2m$ and 153.5.

∴ The correct answer is Ⓐ.

EXAMPLE 4 Real-Life Application

You plant a cypress tree that is 10 inches tall. Each year, its height increases by 15 inches.

a. Make a table that shows the height of the tree for 4 years. Then write an expression for the height after t years.

b. What is the height after 9 years?

a. The height is *increasing*, so *add* 15 each year as shown in the table.

Year, t	Height (inches)
0	10
1	$10 + 15(1) = 25$
2	$10 + 15(2) = 40$
3	$10 + 15(3) = 55$
4	$10 + 15(4) = 70$

When t is 0, the height is 10 inches.

You can see that an expression is $10 + 15t$.

∴ So, the height after year t is $10 + 15t$.

b. Evaluate $10 + 15t$ when $t = 9$.

$$10 + 15t = 10 + 15(9) = 145$$

∴ After 9 years, the height of the tree is 145 inches.

Study Tip

Sometimes, like in Example 3, a variable represents a single value. Other times, like in Example 4, a variable can represent more than one value.

Now You're Ready
Exercises 27–30

On Your Own

7. Your friend has 5 more than twice as many game tokens as your sister. Let t be the number of game tokens your sister has. Write an expression for the number of game tokens your friend has.

8. **WHAT IF?** In Example 4, what is the height of the cypress tree after 16 years?

3.2 Exercises

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Vocabulary and Concept Check

1. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" expressions.

12 more than x	x increased by 12	x take away 12	the sum of x and 12

2. **REASONING** You pay $0.25p$ dollars to print p photos. What does the coefficient represent?

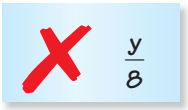
Practice and Problem Solving

Write the phrase as an expression.

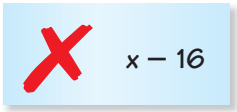
3. 5 less than 8
4. the product of 3 and 12
5. 28 divided by 7
6. the total of 6 and 10
7. 3 fewer than 18
8. 17 added to 15
9. 13 subtracted from a number x
10. 5 times a number d
11. the quotient of 18 and a number a
12. the difference of a number s and 6
13. 7 increased by a number w
14. a number b squared
15. the sum of a number y and 4
16. the difference of 12 and a number x
17. twice a number z
18. a number t cubed

ERROR ANALYSIS **Describe and correct the error in writing the phrase as an expression.**

19. the quotient of 8 and a number y



20. 16 decreased by a number x



21. **DINNER** Five friends share the cost of a dinner equally.
    a. Write an expression for the cost per person.
    b. Make up a total cost and test your expression. Is the result reasonable?

22. **TV SHOW** A television show has 19 episodes per season.
    a. Copy and complete the table.
    b. Write an expression for the number of episodes in n seasons.

Seasons	1	2	3	4	5
Episodes					

Give two ways to write the expression as a phrase.

23. $n + 6$
24. $4w$
25. $15 - b$
26. $14 - 3z$

3 4 Write the phrase as an expression. Then evaluate when $x = 5$ and $y = 20$.

27. 3 less than the quotient of a number y and 4

28. the sum of a number x and 4, all divided by 3

29. 6 more than the product of 8 and a number x

30. the quotient of 40 and the difference of a number y and 16

31. MODELING It costs \$3 to bowl a game and \$2 for shoe rental.

a. Make a table for the cost of up to 5 games.

b. Write an expression for the cost of g games.

c. Use your expression to find the cost of 8 games.

32. PUZZLE Florida has 8 less than 5 times the number of counties in Arizona. Georgia has 25 more than twice the number of counties in Florida.

a. Write an expression for the number of counties in Florida.

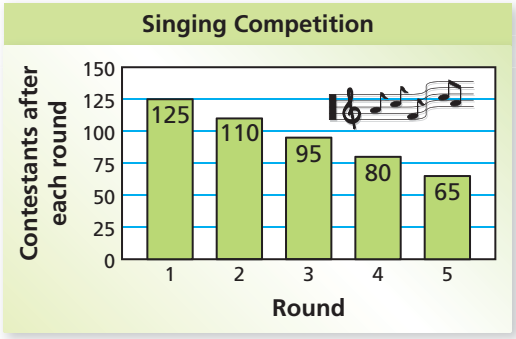
b. Write an expression for the number of counties in Georgia.

c. Arizona has 15 counties. How many do Florida and Georgia have?

33. PATTERNS There are 140 people in a singing competition. The graph shows the results for the first five rounds.

a. Write an expression for the number of people after each round.

b. How many people compete in the ninth round? Explain your reasoning.



34. NUMBER SENSE The difference between two numbers is 8. The lesser number is a. Write an expression for the greater number.

35. Reasoning One number is four times another. The greater number is x. Write an expression for the lesser number.

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Skills Review Handbook)*

36. $8 + (22 + 15)$ **37.** $(13 + 9) + 37$ **38.** $(13 \times 6) \times 5$ **39.** $4 \times (7 \times 5)$

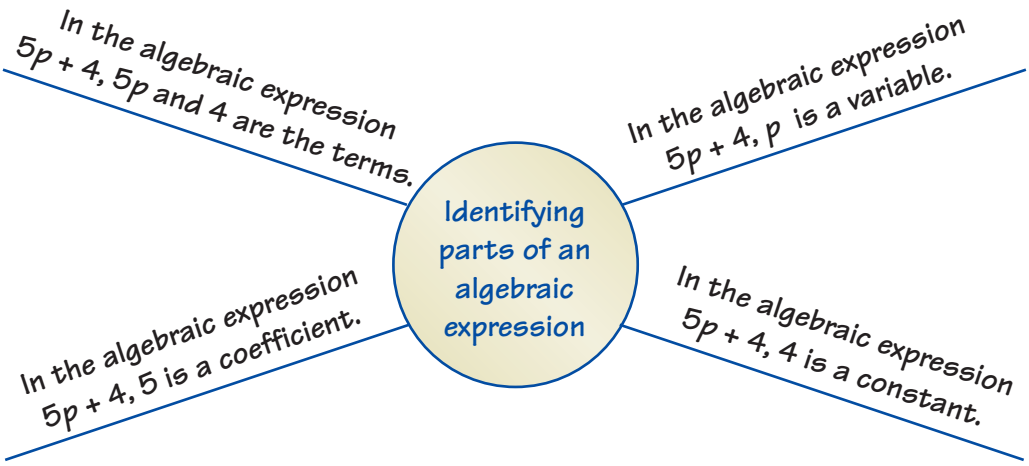
40. MULTIPLE CHOICE A grocery store is making fruit baskets using 144 apples, 108 oranges, and 90 pears. Each basket will be identical. What is the greatest number of fruit baskets the store can make using all the fruit? *(Section 1.5)*

(A) 6 (B) 9 (C) 16 (D) 18

3 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information wheel** to organize information about a topic. Here is an example of an information wheel for identifying parts of an algebraic expression.



On Your Own

Make information wheels to help you study these topics.

1. evaluating algebraic expressions
2. writing algebraic expressions

After you complete this chapter, make information wheels for the following topics.

3. Commutative Properties of Addition and Multiplication
4. Associative Properties of Addition and Multiplication
5. Addition Property of Zero
6. Multiplication Properties of Zero and One
7. Distributive Property
8. factoring expressions

"My information wheel for Fluffy has matching adjectives and nouns."

3.1–3.2 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Identify the terms, coefficients, and constants of the expression. *(Section 3.1)*

1. $6q + 1$

2. $3r^2 + 4r + 8$

Write the expression using exponents. *(Section 3.1)*

3. $s \cdot s \cdot s \cdot s$

4. $2 \cdot t \cdot t \cdot t \cdot t \cdot t$

Evaluate the expression when $a = 8$ and $b = 2$. *(Section 3.1)*

5. $a + 5$

6. ab

7. $a^2 - 6$

Copy and complete the table. *(Section 3.1)*

8.

x	$x + 6$
1	
2	
3	

9.

x	$3x - 5$
3	
6	
9	

Write the phrase as an expression. *(Section 3.2)*

10. the sum of 28 and 35

11. a number x divided by 2

12. the product of a number m and 23

13. 10 less than a number a

14. **COUPON** The expression $p - 15$ is the amount you pay after using the coupon on a purchase of p dollars. How much do you pay for a purchase of \$83? *(Section 3.1)*

15. **AMUSEMENT PARK** The expression $15a + 12c$ is the cost (in dollars) of admission at an amusement park for a adults and c children. Find the total cost for 5 adults and 10 children. *(Section 3.1)*

16. **MOVING TRUCK** To rent a moving truck for the day, it costs \$33 plus \$1 for each mile driven. *(Section 3.2)*

a. Write an expression for the cost to rent the truck.

b. You drive the truck 300 miles. How much do you pay?

3.3 Properties of Addition and Multiplication

Essential Question Does the order in which you perform an operation matter?

1 ACTIVITY: Does Order Matter?

Work with a partner. Place each statement in the correct oval.

a. Fasten 5 shirt buttons.
b. Put on a shirt and tie.
c. Fill and seal an envelope.
d. Floss your teeth.
e. Put on your shoes.
f. Chew and swallow.

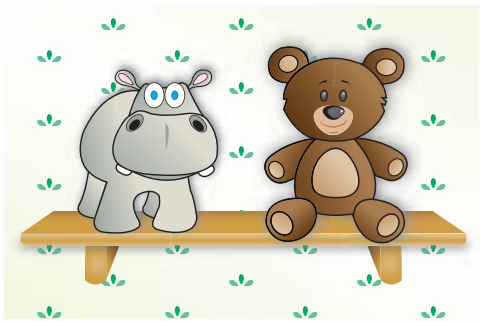
Order Matters

Order Doesn't Matter

Think of three math problems using the four operations where order matters and three where order doesn't matter.

The Meaning of a Word • Commute

When you **commute** the positions of two stuffed animals on a shelf, you switch their positions.



2 ACTIVITY: Commutative Properties

Common Core

Equivalent Expressions

In this lesson, you will
- use properties of operations to generate equivalent expressions.

Learning Standards
6.EE.3
6.EE.4

Work with a partner.

a. Which of the following are true?

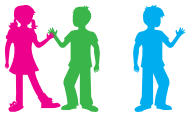
$3 + 5 \stackrel{?}{=} 5 + 3$ $\qquad$ $3 - 5 \stackrel{?}{=} 5 - 3$

$9 \times 3 \stackrel{?}{=} 3 \times 9$ $\qquad$ $9 \div 3 \stackrel{?}{=} 3 \div 9$

b. The true equations show the Commutative Properties of Addition and Multiplication. Why do you think they are called *commutative*?

The Meaning of a Word ● Associate

You have two best friends. Sometimes you **associate** with one of them.



And sometimes you **associate** with the other.

3 ACTIVITY: Associative Properties

Math Practice 3

Use Counterexamples

What do the false equations tell you about the Associative Properties?

Work with a partner.

a. Which of the following are true?

$8 + (3 + 1) \stackrel{?}{=} (8 + 3) + 1$ $\qquad$ $8 - (3 - 1) \stackrel{?}{=} (8 - 3) - 1$

$12 \times (6 \times 2) \stackrel{?}{=} (12 \times 6) \times 2$ $\qquad$ $12 \div (6 \div 2) \stackrel{?}{=} (12 \div 6) \div 2$

b. The true equations show the Associative Properties of Addition and Multiplication. Why do you think they are called *associative*?

What Is Your Answer?

4. IN YOUR OWN WORDS Does the order in which you perform an operation matter? Give examples to support your explanation.

5. MENTAL MATH Explain how you can add the sum in your head.

$$11 + 7 + 12 + 13 + 8 + 9$$

6. SECRET CODE The creatures on a distant planet use the symbols ■, ◆, ★, and ● for the four operations.

a. Use the codes to decide which symbol represents addition and which symbol represents multiplication. Explain your reasoning.

$3 \bullet 4 = 4 \bullet 3$

$3 \bigstar 4 = 4 \bigstar 3$

$2 \bullet (5 \bullet 3) = (2 \bullet 5) \bullet 3$

$2 \bigstar (5 \bigstar 3) = (2 \bigstar 5) \bigstar 3$

$0 \bullet 4 = 0$

$0 \bigstar 4 = 4$

b. Make up your own symbols for addition and multiplication. Write codes using your symbols. Trade codes with a classmate. Decide which symbol represents addition and which symbol represents multiplication.

Practice Use what you learned about the properties of addition and multiplication to complete Exercises 5–8 on page 130.

3.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
equivalent expressions, *p. 128*

Expressions with the same value, like $12 + 7$ and $7 + 12$, are **equivalent expressions**. You can use the Commutative and Associative Properties to write equivalent expressions.

Key Ideas

Commutative Properties

Words Changing the order of addends or factors does not change the sum or product.

Numbers $5 + 8 = 8 + 5$ $\quad$ $5 \cdot 8 = 8 \cdot 5$

Algebra $a + b = b + a$ $\quad$ $a \cdot b = b \cdot a$

Associative Properties

Words Changing the grouping of addends or factors does not change the sum or product.

Numbers $(7 + 4) + 2 = 7 + (4 + 2)$

$(7 \cdot 4) \cdot 2 = 7 \cdot (4 \cdot 2)$

Algebra $(a + b) + c = a + (b + c)$

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

EXAMPLE 1 Using Properties to Write Equivalent Expressions

a. Simplify the expression $7 + (12 + x)$.

$7 + (12 + x) = (7 + 12) + x$ Associative Property of Addition

$= 19 + x$ Add 7 and 12.

b. Simplify the expression $(6.1 + x) + 8.4$.

$(6.1 + x) + 8.4 = (x + 6.1) + 8.4$ Commutative Property of Addition

$= x + (6.1 + 8.4)$ Associative Property of Addition

$= x + 14.5$ Add 6.1 and 8.4.

c. Simplify the expression $5(11y)$.

$5(11y) = (5 \cdot 11)y$ Associative Property of Multiplication

$= 55y$ Multiply 5 and 11.

Study Tip

One way to check whether expressions are equivalent is to evaluate each expression for any value of the variable. In Example 1(a), use $x = 2$.

$7 + (12 + x) = 19 + x$

$7 + (12 + 2) \stackrel{?}{=} 19 + 2$

$21 = 21$ ✓

On Your Own

Now You're Ready
Exercises 5–8

Simplify the expression. Explain each step.

1. $10 + (a + 9)$ **2.** $\left(c + \frac{2}{3}\right) + \frac{1}{2}$ **3.** $5(4n)$

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 Using Properties to Write Equivalent Expressions

a. Simplify the expression $9 \cdot 0 \cdot p$.

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ Associative Property of Multiplication

$= 0 \cdot p = 0$ Multiplication Property of Zero

b. Simplify the expression $4.5 \cdot r \cdot 1$.

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 4.5 \cdot r$ Multiplication Property of One

$= 4.5r$

EXAMPLE 3 Real-Life Application

You and six friends play on a basketball team. A sponsor paid \$100 for the league fee, x dollars for each player's T-shirt, and \$68.25 for trophies. Write an expression for the total amount the sponsor paid.

Common Error

You **and** six friends are on the team, so use the expression $7x$, not $6x$, to represent the cost of the T-shirts.

Add the league fee, the cost of the T-shirts, and the cost of the trophies.

$100 + 7x + 68.25 = 7x + 100 + 68.25$ Commutative Property of Addition

$= 7x + 168.25$ Add 100 and 68.25.

∴ An expression for the total amount is $7x + 168.25$.

On Your Own

Now You're Ready
Exercises 9–23

Simplify the expression. Explain each step.

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

7. WHAT IF? In Example 3, your sponsor paid \$54.75 for trophies. Write an expression for the total amount the sponsor paid.

3.3 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **NUMBER SENSE** Write an example of a sum of fractions. Show that the Commutative Property of Addition is true for the sum.
2. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Addition.
3. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.
4. **WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning.

$7 + (x + 4) = 7 + (4 + x)$

$(3 + b) + 2 = (b + 3) + 2$

$9 + (7 + w) = (9 + 7) + w$

$(4 + n) + 6 = (n + 4) + 6$

Practice and Problem Solving

Tell which property the statement illustrates.

1 5. $5 \cdot p = p \cdot 5$

6. $2 + (12 + r) = (2 + 12) + r$

7. $4 \cdot (x \cdot 10) = (4 \cdot x) \cdot 10$

8. $x + 7.5 = 7.5 + x$

2 9. $(c + 2) + 0 = c + 2$

10. $a \cdot 1 = a$

11. **ERROR ANALYSIS** Describe and correct the error in stating the property that the statement illustrates.

✗ $(7 + x) + 3 = (x + 7) + 3$
Associative Property of Addition

Simplify the expression. Explain each step.

12. $6 + (5 + x)$

13. $(14 + y) + 3$

14. $6(2b)$

15. $7(9w)$

16. $3.2 + (x + 5.1)$

17. $(0 + a) + 8$

18. $9 \cdot c \cdot 4$

19. $(18.6 \cdot d) \cdot 1$

20. $\left(3k + 4\frac{1}{5}\right) + 8\frac{3}{5}$

21. $(2.4 + 4n) + 9$

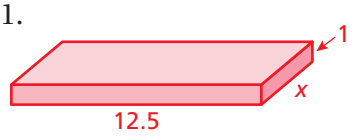
22. $(3s) \cdot 8$

23. $z \cdot 0 \cdot 12$

24. **GEOMETRY** The expression $12 + x + 4$ represents the perimeter of a triangle. Simplify the expression.

25. **RAISIN COOKIES** A case of raisin cookies has 10 cartons. A carton has 12 boxes. The amount you earn on a whole case is $10(12x)$ dollars.
    a. What does x represent?
    b. Simplify the expression.

26. **STRUCTURE** The volume of the rectangular prism is $12.5 \cdot x \cdot 1$.



a. Simplify the expression.

b. Match $x = 0.25$, 12.5, and 144 with the object. Explain.

A. siding for a house **B.** ruler **C.** square floor tile

Write the phrase as an expression. Then simplify the expression.

27. 7 plus the sum of a number x and 5

28. the product of 8 and a number y multiplied by 9

Copy and complete the statement using the specified property.

	Property	Statement
29.	Associative Property of Multiplication	$7(2y) =$
30.	Commutative Property of Multiplication	$13.2 \cdot (x \cdot 1) =$
31.	Associative Property of Addition	$17 + (6 + 2x) =$
32.	Addition Property of Zero	$2 + (c + 0) =$
33.	Multiplication Property of One	$1 \cdot w \cdot 16 =$

34. **HATS** You and a friend sell hats at a fair booth. You sell 16 hats on the first shift and 21 hats on the third shift. Your friend sells x hats on the second shift.

a. Write an expression for the number of hats sold.

b. The expression $37(14) + 10x$ represents the amount that you both earned. How can you tell that your friend was selling the hats for a discounted price?

c. Reasoning You earned more money than your friend. What can you say about the value of x?

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Section 1.3)*

35. $7(10 + 4)$ **36.** $12(10 - 1)$ **37.** $6(5 + 10)$ **38.** $8(30 - 5)$

Find the prime factorization of the number. *(Section 1.4)*

39. 37 **40.** 144 **41.** 147 **42.** 205

43. **MULTIPLE CHOICE** A bag has 16 blue, 20 red, and 24 green marbles. What fraction of the marbles in the bag are blue? *(Skills Review Handbook)*

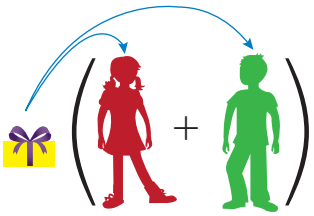
A $\frac{1}{5}$ **B** $\frac{4}{15}$ **C** $\frac{4}{11}$ **D** $\frac{11}{15}$

3.4 The Distributive Property

Essential Question How do you use mental math to multiply two numbers?

The Meaning of a Word Distribute

When you **distribute** something to each person in a group, you give that thing to each person in the group.



1 ACTIVITY: Modeling a Property

Work with a partner.

a. **MODELING** Draw two rectangles of the same width but with different lengths on a piece of grid paper. Label the dimensions.

b. Write an expression for the total area of the rectangles.

$$(\square \times \square) + (\square \times \square)$$

c. Rearrange the rectangles by aligning the shortest sides to form one rectangle. Label the dimensions. Write an expression for the area.

$$\square \times (\square + \square)$$

d. Can the expressions from parts (b) and (c) be set equal to each other? Explain.

e. **REPEATED REASONING** Repeat this activity using different rectangles. Explain how this illustrates the Distributive Property. Write a rule for the Distributive Property.

Common Core

Equivalent Expressions

In this lesson, you will
- use the Distributive Property to find products.
- use the Distributive Property to simplify algebraic expressions.

Learning Standards
6.NS.4
6.EE.2b
6.EE.3
6.EE.4

2 ACTIVITY: Using Mental Math

Math Practice 1

Find Entry Points

How can you rewrite the larger number as the sum of two numbers so that you can use mental math?

Work with a partner. Use the method shown to find the product.

a. Sample: 23×6

$$\begin{array}{rl} 23 & \leftarrow \text{23 is 20 + 3.} \\ \times\ 6 & \\ 120 & \text{Multiply 20 and 6.} \\ +\ 18 & \text{Multiply 3 and 6.} \\ 138 & \text{Add.} \end{array}$$

∴ So, $23 \times 6 = 138$.

b. 33×7

c. 47×9

d. 28×5

e. 17×4

3 ACTIVITY: Using Mental Math

Work with a partner. Use the Distributive Property and mental math to find the product.

a. Sample: 6×23

$$\begin{aligned} 6 \times 23 &= 6 \times (20 + 3) && \text{Write 23 as the sum of 20 and 3.} \\ &= (6 \times 20) + (6 \times 3) && \text{Distribute the 6 over the sum.} \\ &= 120 + 18 && \text{Find the products.} \\ &= 138 && \text{Add.} \end{aligned}$$

∴ So, $6 \times 23 = 138$.

b. 5×17

c. 8×26

d. 20×19

e. 40×29

f. 25×39

g. 15×47

What Is Your Answer?

4. Compare the methods in Activities 2 and 3.

5. IN YOUR OWN WORDS How do you use mental math to multiply two numbers? Give examples to support your explanation.

Practice Use what you learned about the Distributive Property to complete Exercises 5–8 on page 137.

3.4 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
like terms, *p. 136*

Key Idea

Distributive Property

Words To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses. Then evaluate.

Numbers
$3(7 + 2) = 3 \times 7 + 3 \times 2$
$3(7 - 2) = 3 \times 7 - 3 \times 2$

Algebra
$a(b + c) = ab + ac$
$a(b - c) = ab - ac$

EXAMPLE 1 Using Mental Math

Use the Distributive Property and mental math to find 8×53.

$8 \times 53 = 8(50 + 3)$ Write 53 as $50 + 3$.
$= 8(50) + 8(3)$ Distributive Property
$= 400 + 24$ Multiply.
$= 424$ Add.

EXAMPLE 2 Using the Distributive Property

Use the Distributive Property to find $\frac{1}{2} \times 2\frac{3}{4}$.

$\frac{1}{2} \times 2\frac{3}{4} = \frac{1}{2} \times \left(2 + \frac{3}{4}\right)$ Rewrite $2\frac{3}{4}$ as the sum $2 + \frac{3}{4}$.
$= \left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times \frac{3}{4}\right)$ Distributive Property
$= 1 + \frac{3}{8}$ Multiply.
$= 1\frac{3}{8}$ Add.

On Your Own

Now You're Ready
Exercises 5–16

Use the Distributive Property to find the product.

1. 5×41
2. 9×19
3. $6(37)$
4. $\frac{2}{3} \times 1\frac{1}{2}$
5. $\frac{1}{4} \times 4\frac{1}{5}$
6. $\frac{2}{7} \times 3\frac{3}{4}$

EXAMPLE 3 Simplifying Algebraic Expressions

Use the Distributive Property to simplify the expression.

a. $4(n + 5)$

$4(n + 5) = 4(n) + 4(5)$ Distributive Property

$= 4n + 20$ Multiply.

b. $12(2y - 3)$

$12(2y - 3) = 12(2y) - 12(3)$ Distributive Property

$= 24y - 36$ Multiply.

c. $9(6 + x + 2)$

$9(6 + x + 2) = 9(6) + 9(x) + 9(2)$ Distributive Property

$= 54 + 9x + 18$ Multiply.

$= 9x + 54 + 18$ Commutative Property of Addition

$= 9x + 72$ Add 54 and 18.

Study Tip

You can use the Distributive Property when there are more than two terms in the sum or difference.

On Your Own

Now You're Ready
Exercises 17–32

Use the Distributive Property to simplify the expression.

7. $7(a + 2)$ **8.** $3(d - 11)$ **9.** $7(2 + 6 - 4d)$

EXAMPLE 4 Real-Life Application

José is x years old. His brother, Felipe, is 2 years older than José. Their aunt, Maria, is three times as old as Felipe. Write and simplify an expression that represents Maria's age in years.

Name	Description	Expression
José	He is x years old.	x
Felipe	He is 2 years *older* than José. So, *add* 2 to x.	$x + 2$
Maria	She is three *times* as old as Felipe. So, *multiply* 3 and $(x + 2)$.	$3(x + 2)$

$3(x + 2) = 3(x) + 3(2)$ Distributive Property

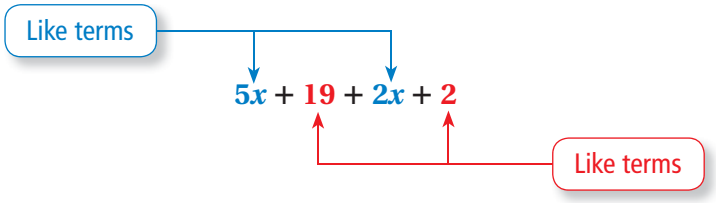
$= 3x + 6$ Multiply.

∴ Maria's age in years is represented by the expression $3x + 6$.

On Your Own

10. Alexis is x years old. Her sister, Gloria, is 7 years older than Alexis. Their grandfather is five times as old as Gloria. Write and simplify an expression that represents their grandfather's age in years.

In an algebraic expression, **like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.



Use the Distributive Property to *combine* like terms.

EXAMPLE 5 Combining Like Terms

Simplify each expression.

a. $3x + 9 + 2x - 5$

$3x + 9 + 2x - 5 = 3x + 2x + 9 - 5$ Commutative Property of Addition

$= (3 + 2)x + 9 - 5$ Distributive Property

$= 5x + 4$ Simplify.

b. $y + y + y$

$y + y + y = 1y + 1y + 1y$ Multiplication Property of One

$= (1 + 1 + 1)y$ Distributive Property

$= 3y$ Add coefficients.

c. $7z + 2(z - 5y)$

$7z + 2(z - 5y) = 7z + 2(z) - 2(5y)$ Distributive Property

$= 7z + 2z - 10y$ Multiply.

$= (7 + 2)z - 10y$ Distributive Property

$= 9z - 10y$ Add coefficients.

On Your Own

Now You're Ready
Exercises 39–53

Simplify the expression.

11. $8 + 3z - z$ **12.** $3(b + 5) + b + 2$

3.4 Exercises

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Vocabulary and Concept Check

1. **WRITING** One meaning of the word *distribute* is "to give something to each member of a group." How can this help you remember the Distributive Property?
2. **OPEN-ENDED** Write an algebraic expression in which you use the Distributive Property and then the Associative Property of Addition to simplify.
3. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$2(x + 2)$ $5(x - 8)$ $4 + (x \cdot 4)$ $8(9 - x)$

4. Identify the like terms in the expression $8x + 1 + 7x + 4$.

Practice and Problem Solving

Use the Distributive Property and mental math to find the product.

1 **5.** 3×21 **6.** 9×76 **7.** $12(43)$ **8.** $5(88)$

9. 18×52 **10.** 8×27 **11.** $8(63)$ **12.** $7(28)$

Use the Distributive Property to find the product.

2 **13.** $\frac{1}{4} \times 2\frac{2}{7}$ **14.** $\frac{5}{6} \times 2\frac{2}{5}$ **15.** $\frac{5}{9} \times 4\frac{1}{2}$ **16.** $\frac{2}{15} \times 5\frac{5}{8}$

Use the Distributive Property to simplify the expression.

3 **17.** $3(x + 4)$ **18.** $10(b - 6)$ **19.** $6(s - 9)$ **20.** $7(8 + y)$

21. $8(12 + a)$ **22.** $9(2n + 1)$ **23.** $12(6 - k)$ **24.** $18(5 - 3w)$

25. $9(3 + c + 4)$ **26.** $7(8 + x + 2)$ **27.** $8(5g + 5 - 2)$ **28.** $6(10 + z + 3)$

29. $4(x + y)$ **30.** $25(x - y)$ **31.** $7(p + q + 9)$ **32.** $13(n + 4 + 7m)$

33. ERROR ANALYSIS Describe and correct the error in rewriting the expression.

✗ $6(y + 8) = 6y + 8$

34. ART MUSEUM A class of 30 students visits an art museum and a special exhibit while there.

a. Use the Distributive Property to write and simplify an expression for the cost.

b. Estimate a reasonable value for x. Explain.

c. Use your estimate for x to evaluate the original expression and the simplified expression in part (a). Are the values the same?

35. FITNESS Each day, you run on a treadmill for r minutes and lift weights for 15 minutes. Which expressions can you use to find how many minutes of exercise you do in 5 days? Explain your reasoning.

$5(r + 15)$ $\quad 5r + 5 \cdot 15$ $\quad 5r + 15$ $\quad r(5 + 15)$

36. SPEED A cheetah can run 103 feet per second. A zebra can run x feet per second. Use the Distributive Property to write and simplify an expression for how much farther the cheetah can run in 10 seconds.

UNIFORMS Your baseball team has 16 players. Use the Distributive Property to write and simplify an expression for the total cost of buying the items shown for all the players.

37.

Pants: \$10 $\quad$ Belt: \$$x$

38.

Jersey: \$12 $\quad$ Socks: \$4 $\quad$ Hat: \$$x$

5 Simplify the expression.

39. $6(x + 4) + 1$ $\quad$ **40.** $5 + 8(3 + x)$ $\quad$ **41.** $7(8 + 4k) + 12$

42. $x + 3 + 5x$ $\quad$ **43.** $7y + 6 - 1 + 12y$ $\quad$ **44.** $w + w + 5w$

45. $4d + 9 - d - 8$ $\quad$ **46.** $n + 3(n - 1)$ $\quad$ **47.** $2v + 8v - 5v$

48. $5(z + 4) + 5(2 - z)$ $\quad$ **49.** $2.7(w - 5.2)$ $\quad$ **50.** $\frac{2}{3}y + \frac{1}{6}y + y$

51. $\frac{3}{4}\left(z + \frac{2}{5}\right) + 2z$ $\quad$ **52.** $7(x + y) - 7x$ $\quad$ **53.** $4x + 9y + 3(x + y)$

54. ERROR ANALYSIS Describe and correct the error in simplifying the expression.

$$8x - 2x + 5x = 8x - 7x$$
$$= (8 - 7)x$$
$$= x$$

ALGEBRA Find the value of x that makes the expressions equivalent.

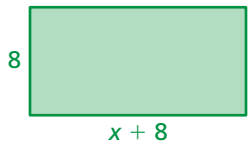
55. $4(x - 5);\ 32 - 20$ $\quad$ **56.** $2(x + 9);\ 30 + 18$ $\quad$ **57.** $7(8 - x);\ 56 - 21$

58. REASONING Simplify the expressions and compare. What do you notice? Explain.

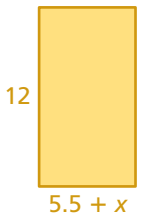
$4(x + 6)$ $\quad (x + 6) + (x + 6) + (x + 6) + (x + 6)$

GEOMETRY **Write and simplify expressions for the area and perimeter of the rectangle.**

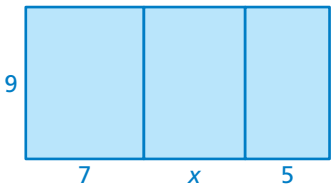
59.



60.



61.



62. **FUNDRAISER** An art club sells 42 large candles and 56 small candles.

a. Use the Distributive Property to write and simplify an expression for the profit.

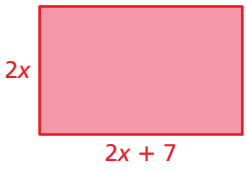
b. A large candle costs \$5, and a small candle costs \$3. What is the club's profit?

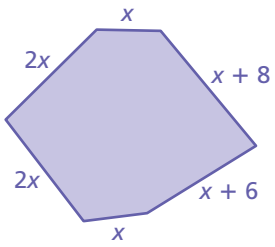
Profit = Price − Cost

63. **REASONING** Evaluate each expression by (1) using the Distributive Property and (2) evaluating inside the parentheses first. Which method do you prefer? Is your preference the same for both expressions? Explain your reasoning.

a. $2(3.22 - 0.12)$ **b.** $12\left(\frac{1}{2} + \frac{2}{3}\right)$

64. **REASONING** Write and simplify an expression for the difference between the perimeters of the rectangle and the hexagon. Interpret your answer.





65. **Puzzle** Add one set of parentheses to the expression $7 \cdot x + 3 + 8 \cdot x + 3 \cdot x + 8 - 9$ so that it is equivalent to $2(9x + 10)$.

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Section 2.4, Section 2.5, and Section 2.6)*

66. $4.871 + 7.4 - 1.63$

67. $25.06 - 0.049 + 8.995$

68. $15.3 \cdot 9.1 - 4.017$

69. $29.24 \div 3.4 \cdot 0.045$

70. **MULTIPLE CHOICE** What is the GCF of 48, 80, and 96? *(Section 1.5)*

(A) 12 (B) 16

(C) 24 (D) 480

Extension 3.4 Factoring Expressions

Check It Out
Lesson Tutorials
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Key Vocabulary
factoring an expression, *p. 140*

Key Idea

Factoring an Expression

Words Writing a numerical expression or algebraic expression as a product of factors is called **factoring the expression**. You can use the Distributive Property to factor expressions.

Numbers $3 \cdot 7 + 3 \cdot 2 = 3(7 + 2)$
$3 \cdot 7 - 3 \cdot 2 = 3(7 - 2)$

Algebra $ab + ac = a(b + c)$
$ab - ac = a(b - c)$

EXAMPLE 1 Factoring a Numerical Expression

Study Tip
When you factor an expression, you can *factor out* any common factor.

Factor $20 - 12$ using the GCF.

Find the GCF of 20 and 12 by listing their factors.

Factors of 20: ①,②,④, 5, 10, 20
Factors of 12: ①,②, 3,④, 6, 12

Circle the common factors.

The GCF of 20 and 12 is 4.

Write each term of the expression as a product of the GCF and the remaining factor. Then use the Distributive Property to factor the expression.

$20 - 12 = 4(5) - 4(3)$ Rewrite using GCF.
$= 4(5 - 3)$ Distributive Property

EXAMPLE 2 Identifying Equivalent Expressions

Which expression is not equivalent to $16x + 24$?

Ⓐ $2(8x + 12)$ Ⓑ $4(4x + 6)$ Ⓒ $6(3x + 4)$ Ⓓ $(2x + 3)8$

Common Core
Equivalent Expressions
In this extension, you will
- use the Distributive Property to produce equivalent expressions.

Learning Standards
6.NS.4
6.EE.3
6.EE.4

Each choice is a product of two factors in which one is a whole number and the other is the sum of two terms. For an expression to be equivalent to $16x + 24$, its whole number factor must be a common factor of 16 and 24.

Factors of 16: ①,②,④,⑧, 16
Factors of 24: ①,②, 3,④, 6,⑧, 12, 24

Circle the common factors.

The common factors of 16 and 24 are 1, 2, 4, and 8. Because 6 is not a common factor of 16 and 24, Choice C cannot be equivalent to $16x + 24$.

Check: $6(3x + 4) = 6(3x) + 6(4) = 18x + 24 \neq 16x + 24$ ✗

∴ So, the correct answer is Ⓒ.

EXAMPLE 3 Factoring an Algebraic Expression

You receive a discount on each book you buy for your electronic reader. The original price of each book is x dollars. You buy 5 books for a total of $(5x - 15)$ dollars. Factor the expression. What can you conclude about the discount?

Find the GCF of $5x$ and 15 by writing their prime factorizations.

$5x = 5 \cdot x$

$15 = 5 \cdot 3$ Circle the common prime factor.

So, the GCF of $5x$ and 15 is 5. Use the GCF to factor the expression.

$5x - 15 = 5(x) - 5(3)$ Rewrite using GCF.

$= 5(x - 3)$ Distributive Property

The factor 5 represents the number of books purchased. The factor $(x - 3)$ represents the price of each book. This factor is a difference of two terms, showing that the price x of each book is decreased by \$3.

So, the factored expression shows a \$3 discount for every book you buy. The original expression shows a total savings of \$15.

Practice

Factor the expression using the GCF.

1. $7 + 14$ **2.** $44 - 11$ **3.** $18 - 12$ **4.** $70 + 95$

5. $60 - 36$ **6.** $100 - 80$ **7.** $84 + 28$ **8.** $48 + 80$

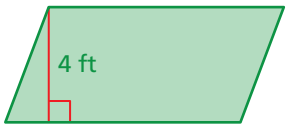
9. $2x + 10$ **10.** $15x + 6$ **11.** $26x - 13$ **12.** $50x - 60$

13. $36x + 9$ **14.** $14x - 98$ **15.** $10x - 25y$ **16.** $24y + 88x$

17. REASONING The whole numbers a and b are divisible by c. Is $a + b$ divisible by c? Is $b - a$ divisible by c? Explain your reasoning.

18. OPEN-ENDED Write five expressions that are equivalent to $8x + 16$.

19. GEOMETRY The area of the parallelogram is $(4x + 16)$ square feet. Write an expression for the base.



20. STRUCTURE You buy 37 concert tickets for \$8 each, and then sell all 37 tickets for \$11 each. The work below shows two ways you can determine your profit. Describe each solution method. Which do you prefer? Explain your reasoning.

profit $= 37(11) - (37)8$
$= 407 - 296$
$= \$111$

profit $= 37(11) - (37)8$
$= 37(11 - 8)$
$= 37(3)$
$= \$111$

3.3–3.4 Quiz

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Progress Check
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Tell which property the statement illustrates. *(Section 3.3)*

1. $3.5 \cdot z = z \cdot 3.5$

2. $14 + (35 + w) = (14 + 35) + w$

Simplify the expression. Explain each step. *(Section 3.3)*

3. $3.2 + (b + 5.7)$

4. $6 \cdot (10 \cdot k)$

Use the Distributive Property and mental math to find the product. *(Section 3.4)*

5. 6×49

6. 7×86

Use the Distributive Property to simplify the expression. *(Section 3.4)*

7. $5(x - 8)$

8. $7(y + 3)$

Simplify the expression. *(Section 3.4)*

9. $6q + 2 + 3q + 5$

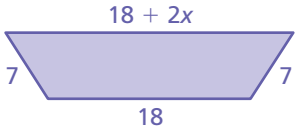
10. $4r + 3(r - 2)$

Factor the expression using the GCF. *(Section 3.4)*

11. $12 + 21$

12. $16x - 36$

13. GEOMETRY The expression $18 + 7 + (18 + 2x) + 7$ represents the perimeter of the trapezoid. Simplify the expression. *(Section 3.3)*



Movie Tickets	Snacks
Student $8	Candy $3
Adult $10	Popcorn $x

14. MOVIES You and four of your friends go to a movie and each buy popcorn. *(Section 3.4)*

a. Use the Distributive Property to write an expression for the total cost to buy movie tickets and popcorn. Simplify the expression.

b. Choose a reasonable value for x. Evaluate the expression.

15. GEOMETRY The length of a rectangle is 16 inches, and its area is $(32x + 48)$ square inches. Factor the expression for the area. Write an expression for the width. *(Section 3.4)*

3 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

algebraic expression, *p. 112*
terms, *p. 112*
variable, *p. 112*
coefficient, *p. 112*
constant, *p. 112*
equivalent expressions, *p. 128*
like terms, *p. 136*
factoring an expression, *p. 140*

Review Examples and Exercises

3.1 Algebraic Expressions *(pp. 110–117)*

a. Evaluate $a \div b$ when $a = 48$ and $b = 8$.

$a \div b = 48 \div 8$ — Substitute 48 for a and 8 for b.

$= 6$ — Divide 48 by 8.

b. Evaluate $y^2 - 14$ when $y = 5$.

$y^2 - 14 = 5^2 - 14$ — Substitute 5 for y.

$= 25 - 14$ — Using order of operations, evaluate 5^2.

$= 11$ — Subtract 14 from 25.

Exercises

Evaluate the expression when $x = 20$ and $y = 4$.

1. $x \div 5$ **2.** $y + x$ **3.** $8y - x$

4. GAMING In a video game, you score p game points and b triple bonus points. An expression for your score is $p + 3b$. What is your score when you earn 245 game points and 20 triple bonus points?

3.2 Writing Expressions *(pp. 118–123)*

Write the phrase as an expression.

a. a number z decreased by 18

$z - 18$ — The phrase *decreased by* means *subtraction*.

b. the sum of 7 and the product of a number x and 12

$7 + 12x$ — The phrase *sum of* means *addition*.
The phrase *product of* means *multiplication*.

Exercises

Write the phrase as an expression.

5. 11 fewer than a number b

6. the product of a number d and 32

7. 18 added to a number n

8. a number t decreased by 17

9. BASKETBALL Your basketball team scored 4 fewer than twice as many points as the other team.

a. Write an expression for the number of points your team scored.

b. The other team scored 24 points. How many points did your team score?

3.3 Properties of Addition and Multiplication *(pp. 126–131)*

a. Simplify the expression $(x + 18) + 4$.

$(x + 18) + 4 = x + (18 + 4)$ Associative Property of Addition

$= x + 22$ Add 18 and 4.

b. Simplify the expression $(5.2 + a) + 0$.

$(5.2 + a) + 0 = 5.2 + (a + 0)$ Associative Property of Addition

$= 5.2 + a$ Addition Property of Zero

c. Simplify the expression $36 \cdot r \cdot 1$.

$36 \cdot r \cdot 1 = 36 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 36 \cdot r$ Multiplication Property of One

$= 36r$

Exercises

Simplify the expression. Explain each step.

10. $10 + (2 + y)$

11. $(21 + b) + 1$

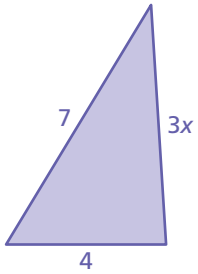
12. $3(7x)$

13. $1(3.2w)$

14. $5.3 + (w + 1.2)$

15. $(0 + t) + 9$

16. GEOMETRY The expression $7 + 3x + 4$ represents the perimeter of the triangle. Simplify the expression.



3.4 The Distributive Property *(pp. 132–141)*

a. Use the Distributive Property to simplify $3(n + 9)$.

$3(n + 9) = 3(n) + 3(9)$ Distributive Property

$= 3n + 27$ Multiply.

b. Simplify $5x + 7 + 3x - 2$.

$5x + 7 + 3x - 2 = 5x + 3x + 7 - 2$ Commutative Property of Addition

$= (5 + 3)x + 7 - 2$ Distributive Property

$= 8x + 5$ Simplify.

c. Factor $14x - 49$ using the GCF.

Find the GCF of $14x$ and 49 by writing their prime factorizations.

$14x = 2 \cdot 7 \cdot x$ Circle the common prime factor.

$49 = 7 \cdot 7$

So, the GCF of $14x$ and 49 is 7. Use the GCF to factor the expression.

$14x - 49 = 7(2x) - 7(7)$ Rewrite using GCF.

$= 7(2x - 7)$ Distributive Property

Exercises

Use the Distributive Property to find the product.

17. $\frac{3}{4} \times 2\frac{1}{3}$ **18.** $\frac{4}{7} \times 4\frac{5}{8}$ **19.** $\frac{1}{5} \times 5\frac{10}{11}$

Use the Distributive Property to simplify the expression.

20. $2(x + 12)$ **21.** $11(b - 3)$ **22.** $8(s - 1)$

23. $6(6 + y)$ **24.** $25(z - 4)$ **25.** $35(w - 2)$

26. HAIRCUT A family of four goes to a salon for haircuts. The cost of each haircut is \$13. Use the Distributive Property and mental math to find the product 4×13 for the total cost.

Simplify the expression.

27. $5(n + 3) + 4n$ **28.** $t + 2 + 6t$ **29.** $3z + 4 + 5z - 9$

Factor the expression using the GCF.

30. $15 + 35$ **31.** $36x - 28$ **32.** $16x + 56y$

3 Chapter Test

Check It Out
Test Practice
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Evaluate the expression when $a = 6$ and $b = 8$.

1. $4 + a$ **2.** $a - 6$ **3.** ab

Write the phrase as an expression.

4. twice a number x **5.** 25 more than 50 **6.** 40 divided by 5

Simplify the expression. Explain each step.

7. $3.1 + (8.6 + m)$ **8.** $(10 \cdot n) \cdot 7$ **9.** $3(15w)$

Use the Distributive Property to simplify the expression.

10. $4(x + 8)$ **11.** $12(y - 5)$

Simplify the expression.

12. $4(q + 2) - 6$ **13.** $3(2 + 5r) + 11$

14. $s + 3s + 4s$ **15.** $4t - 2 - 2t + 7$

Factor the expression using the GCF.

16. $18 + 24$ **17.** $40 - 16$

18. $15x + 20$ **19.** $32x - 40y$

20. SOCCER GAME Playing time is added at the end of a soccer game to make up for stoppages. An expression for the length of a 90-minute soccer game with x minutes of stoppage time is $90 + x$. How long is a game with 4 minutes of stoppage time?

21. GEOMETRY The expression $15 \cdot x \cdot 6$ represents the volume of a rectangular prism with a length of 15, a width of x, and a height of 6. Simplify the expression.

22. PARTY FAVORS You make party favors for an event. You tie 9 inches of ribbon around each party favor. Write an expression for the amount of ribbon you need for n party favors. The ribbon costs \$3 for each *yard*. Write an expression for the total cost of the ribbon.

3 Standards Assessment

Test-Taking Strategy

After Answering Easy Questions, Relax

1. The student council is organizing a school fair. Council members are making signs to show the prices for admission and for each game a person can play.

SCHOOL FAIR	
Admission	$2.00
Price per game	$0.25

Let x represent the number of games. Which expression can you use to determine the total amount, in dollars, a person pays for admission and playing x games? *(6.EE.2a)*

A. 2.25

B. $2.25x$

C. $2 + 0.25x$

D. $2x + 0.25$

2. Which property does the equation below represent? *(6.EE.3)*

$$17 \cdot 44 + 17 \cdot 56 = 17 \cdot 100$$

F. Distributive Property

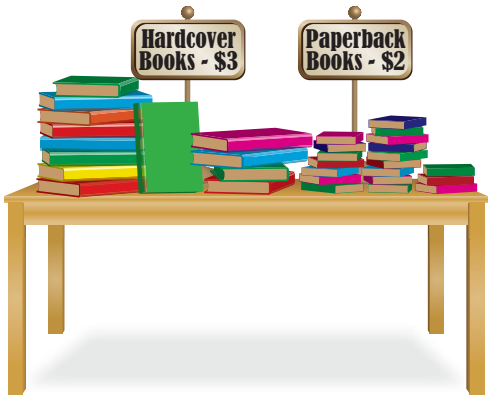
G. Multiplication Property of One

H. Associative Property of Multiplication

I. Commutative Property of Multiplication

3. At a used book store, you can purchase two types of books.

You can use the expression $3h + 2p$ to find the total cost for h hardcover books and p paperback books. What is the total cost, in dollars, for 6 hardcover books and 4 paperback books? *(6.EE.2c)*



4. What is the value of 9.6×12.643? *(6.NS.3)*

A. 12.13728

B. 121.3728

C. 1213.728

D. 12,137.28

5. What is the value of 4.391 + 5.954? *(6.NS.3)*

F. 9.12145

G. 9.245

H. 9.345

I. 10.345

6. Which number pair has a greatest common factor of 6? *(6.NS.4)*

A. 18, 54

B. 30, 42

C. 30, 60

D. 36, 60

7. Properties of Addition and Multiplication are used to simplify an expression.

$$\begin{aligned} 36 \cdot 23 + 33 \cdot 64 &= 36 \cdot 23 + 64 \cdot 33 \\ &= 36 \cdot 23 + 64 \cdot (23 + 10) \\ &= 36 \cdot 23 + 64 \cdot 23 + 64 \cdot 10 \\ &= x \cdot 23 + 64 \cdot 10 \\ &= 2300 + 640 \\ &= 2940 \end{aligned}$$

What number belongs in place of the x? *(6.EE.3)*

8. Which property was used to simplify the expression? *(6.EE.3)*

$$\begin{aligned} (47 \times 125) \times 8 &= 47 \times (125 \times 8) \\ &= 47 \times 1000 \\ &= 47{,}000 \end{aligned}$$

F. Distributive Property

G. Multiplication Property of One

H. Associative Property of Multiplication

I. Commutative Property of Multiplication

9. What is the value of the expression below when $a = 5$, $b = 7$, and $c = 6$? *(6.EE.2c)*

$$9b - 4a + 2c$$

A. 29

B. 31

C. 55

D. 78

10. Which equation correctly demonstrates the Distributive Property? *(6.EE.4)*

F. $a(b + c) = ab + c$

G. $a(b + c) = ab + ac$

H. $a + (b + c) = (a + b) + (a + c)$

I. $a + (b + c) = (a + b) \cdot (a + c)$

11. Which expression is equivalent to $3\frac{3}{5} \div 6\frac{1}{2}$? *(6.NS.1)*

A. $\frac{5}{18} \times \frac{13}{2}$

B. $\frac{18}{5} \times \frac{2}{13}$

C. $\frac{9}{5} \div \frac{6}{2}$

D. $\frac{18}{5} \div \frac{2}{13}$

12. Which number pair does *not* have a least common multiple of 24? *(6.NS.4)*

F. 2, 12

G. 3, 8

H. 6, 8

I. 12, 24

13. Think Solve Explain Use the Properties of Multiplication to simplify the expression in an efficient way. Show your work and explain how you used the Properties of Multiplication. *(6.EE.3)*

$$(25 \times 18) \times 4$$

14. You evaluated an expression using $x = 6$ and $y = 9$. You correctly got an answer of 105. Which expression did you evaluate? *(6.EE.2c)*

A. $3x + 6y$

B. $5x + 10y$

C. $6x + 9y$

D. $10x + 5y$

15. Which number is equivalent to the expression below? *(6.EE.1)*

$$2 \times 12 - 8 \div 2^2$$

F. 2

G. 4

H. 8

I. 22